Introduction:

Probability calculus, classical / frequentist and Bayesian statistics

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What *is* probability?

- Chevalier de Méré,Blaise Pascal, Pierre Fermat, and gambling problems mid-1600s
  - Combinatorics & arithmetics (dice tossing etc.)
- Jacob Bernoulli and statistical probability 1713
  - Law of large numbers

➢ *Classical probability*
  - Frequence interpretation (statistical probability)

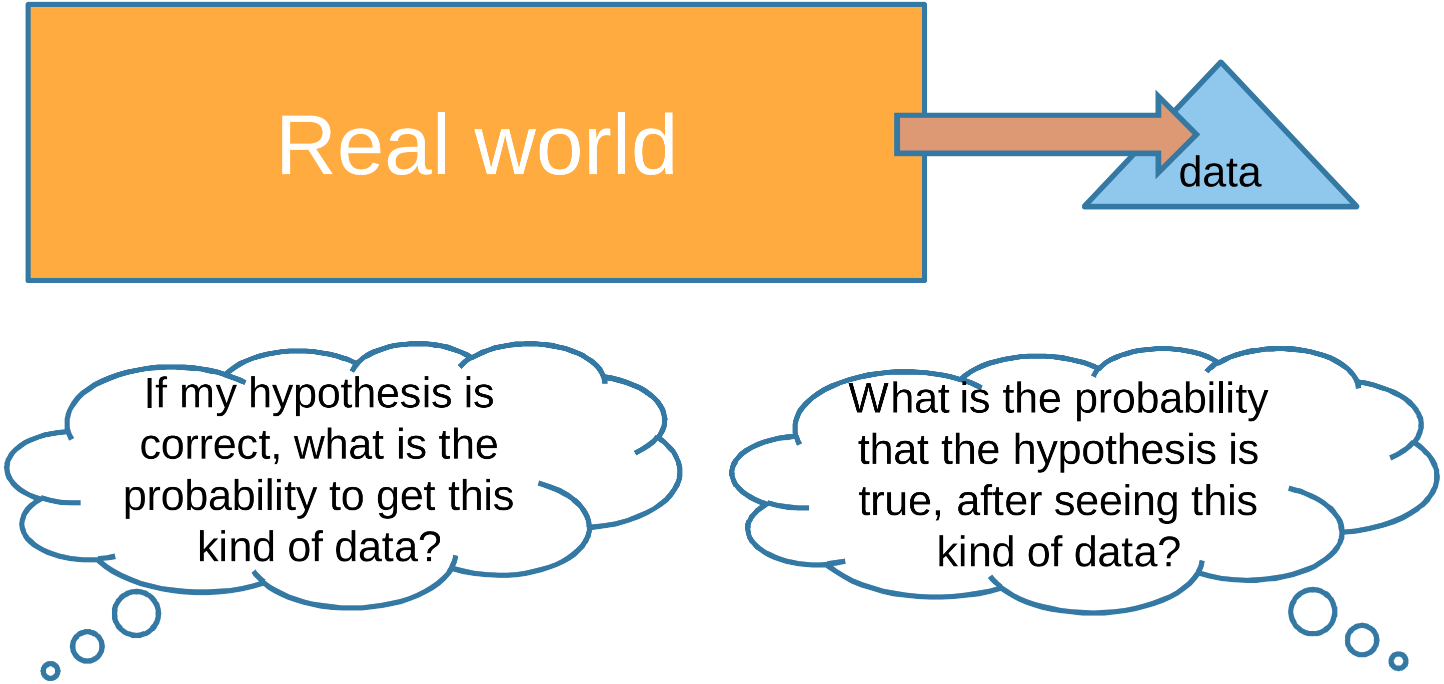
- Axiomatization of probability: Kolmogorov 1933
  - P≥0
  - Non-overlapping probabilities sum up
  - Sum of probabilities of all possiblities = 1

Bayesian probability

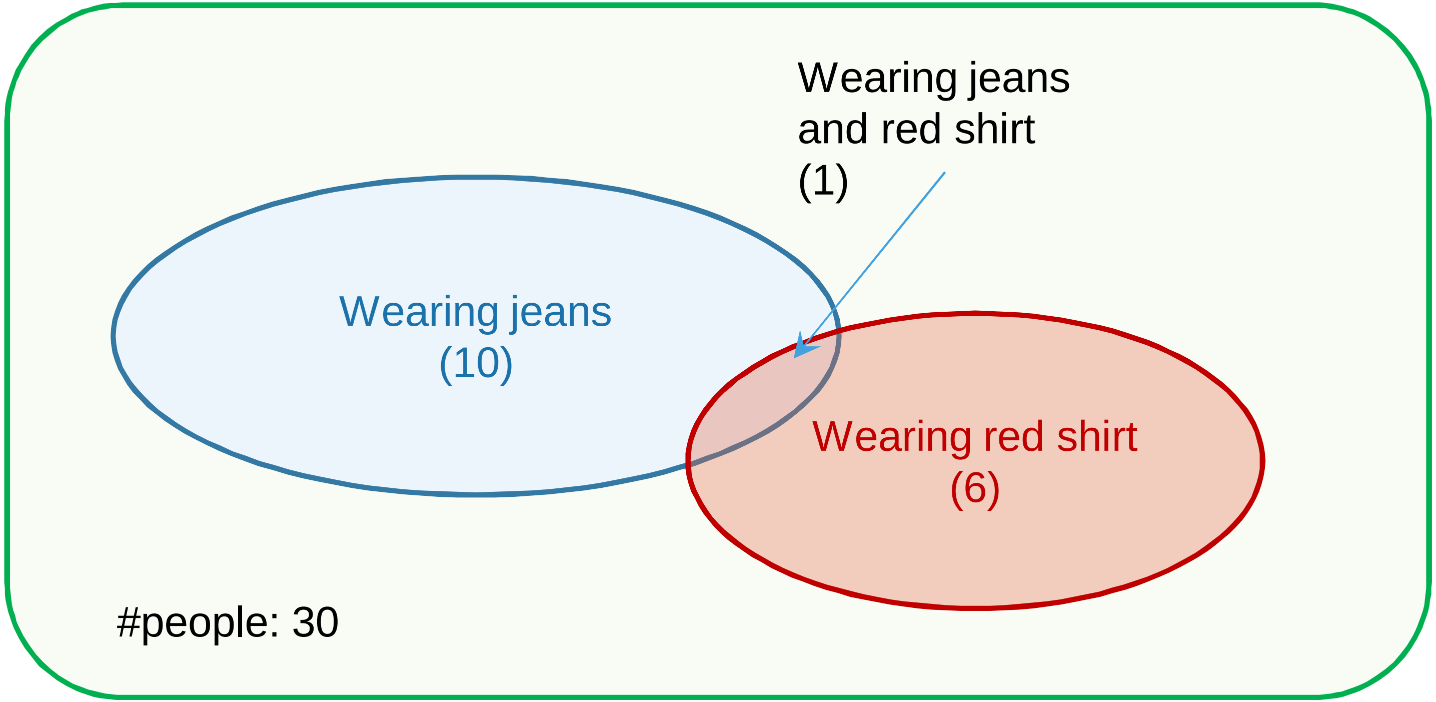
- Bayesian probability: The *credibility* of events
  - Objective interpretation: **a reasonable credibility, given all the information that is available** (*a robot would come to this conclusion*)
  - Subjective interpretation: **(subjective) degree of belief, given the available evidence**
  - The difference is mostly philosophical and technical

- Thomas Bayes 1700s, Laplace
- A renaissance of Bayesian statistics after the computation power increased since 1980s

How do Bayesian and frequentist interpretations differ?

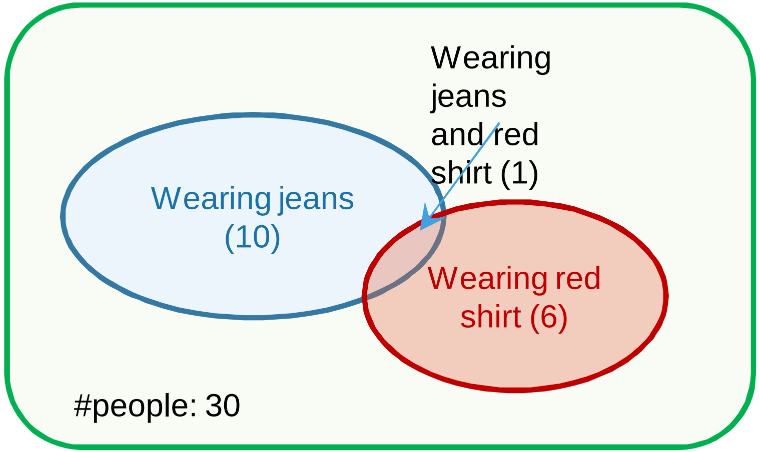


Bayes theorem



Bayes theorem

- What is the probability that a randomly selected person is wearing jeans?
  - P(A) = 10/30 = 0.333
- P(red shirt) = P(B) = 6/30 =0.2

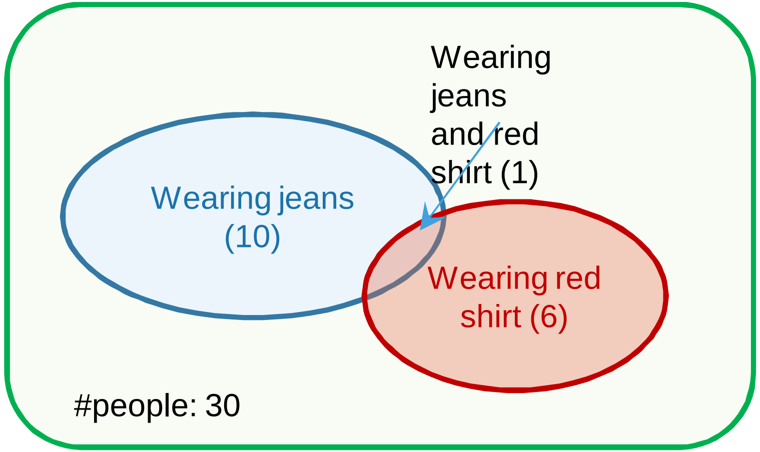


- What is the probability that a person is wearing a red shirt, **if they are also wearing jeans**?
  - P(B|A) = 1/10 = 0.1

Conditional probability!

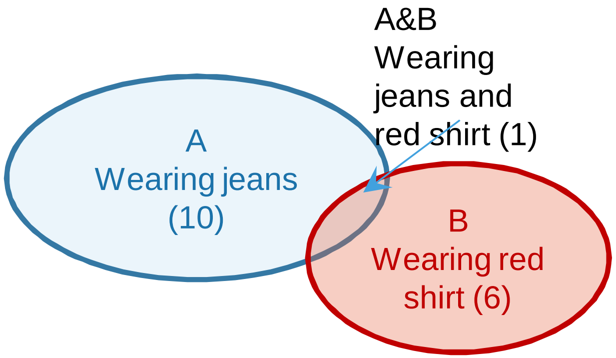
Bayes theorem

- What is the probability that a person is wearing jeans **and** a red shirt?
  - Trivially, P(A&B) = 1 / 30 = 0.0333

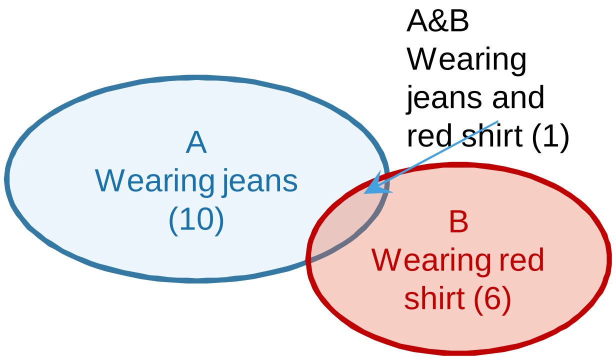


Bayes theorem

- The same probability can be calculated using conditional probabilities:
  - P(A&B)=P(A)P(B|A) = 10/30 * 1/10 = 1/30 = 0.033



- Or the other way around:
  - P(A&B)=P(B)P(A|B) = 6/30*1/6 = 6/180 = 1/30 = 0.033



Bayes theorem

- We have shown that

$$P(A)P(B|A) = P(B)P(A|B)$$

- Rearrange

$$P(B|A) = \frac{P(B)P(A|B)}{P(A)}$$

- Change A and B into data and model!

This is Bayes theorem!

Bayesian vs. classical statistics (recap)

- In the classical world view, it does not make sense to ask for the probability that a hypothesis (or model) is true
    - It is or it is not – unique events don't have probability!

- Bayesian interpretation of probability: *"How likely / credible it is, in the light of the best available knowledge, that the hypothesis (model) is true?"*